Chapter (9) Trigonometry

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1. (a) Solve $\sin x \cos x = 0.5 \tan x$ for $0° \le x \le 180°$. [3]

$$\sin x \cos x = \frac{1}{2}\frac{\sin x}{\cos x}$$

$$\sin x \cos x - \frac{1}{2}\frac{\sin x}{\cos x} = 0$$

$$\frac{2\sin x \cos^2 x - \sin x}{2\cos x} = 0$$

$$\frac{\sin x}{2\cos x} \times (2\cos^2 x - 1) = 0$$

$\frac{1}{2}\tan x = 0$ or $2\cos^2 x - 1 = 0$

$\tan x = 0$

$x = \tan^{-1}(0)$

$= 0, 180° + 0$

$= 0, 180°$

S | A
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$2\cos^2 x = 1$

$\cos x = \pm \frac{1}{\sqrt{2}}$

$x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= 45°$

For negative,

$x = 180° - 45$

$= 135°$

(b) (i) Show that $sec\,\theta - \frac{sin\theta}{cot\,\theta} = cos\,\theta$. [3]

$$L.H.S = \frac{1}{\cos\theta} - \sin\theta \times \tan\theta$$

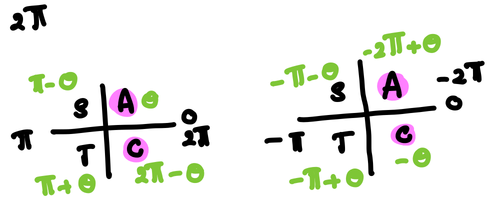
$$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$$

$$= R.H.S$$

(ii) Hence solve $sec\,3\theta - \frac{sin\,3\theta}{cot\,3\theta} = 0.5$ for $-\frac{2\pi}{3} \leq \theta \leq \frac{2\pi}{3}$, where θ is in radians,

$$\cos 3\theta = \frac{1}{2} \qquad -2\pi \leqslant 3\theta \leqslant 2\pi$$



$$3\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$3\theta = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3},\ -\frac{\pi}{3},\ -2\pi + \frac{\pi}{3}$$

$$= \frac{\pi}{3},\ \frac{5\pi}{3},\ -\frac{\pi}{3},\ -\frac{5\pi}{3}$$

$$\theta = \frac{\pi}{9},\ \frac{5\pi}{9},\ -\frac{\pi}{9},\ -\frac{5\pi}{9}$$

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2. (a) (i) Show that $sec\,\theta - \frac{tan\,\theta}{cose\theta} = cos\theta$. [3]

$$L.H.S = \frac{1}{\cos\theta} - \tan\theta \times \sin\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \times \sin\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$$

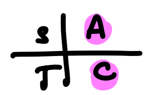
$$= R.H.S$$

(ii) Solve $sec\,2\theta - \frac{tan\,2\theta}{cosec\,2\theta} = \frac{\sqrt{3}}{2}$ for $0° \le \theta \le 180°$. [3]

$0° \le 2\theta \le 360°$

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$



$$= 30°,\ 360° - 30°$$

$$= 30°,\ 330°$$

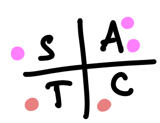
$$\theta = 15°,\ 165°$$

(b) Solve $2sin^2(\phi + \frac{\pi}{3}) = 1$ for $0 \le \phi \le 2\pi\,radians$. [4]

$$\sin\left(\phi + \frac{\pi}{3}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{3} \le \phi + \frac{\pi}{3} \le \frac{7\pi}{3}$$

$$\phi + \frac{\pi}{3} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$



$$= \frac{\pi}{4},\ \pi - \frac{\pi}{4},\ 2\pi + \frac{\pi}{4},$$

$$\pi + \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$$

$$= \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{9\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$$

$$\phi = -\frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{23\pi}{12},\ \frac{11\pi}{12},\ \frac{17\pi}{12}$$

(reject)

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3. (a) (i) Show that $\frac{cosec\,\theta - cot\,\theta}{sin\,\theta} = \frac{1}{1+cos\,\theta}$. [4]

$$L.H.S \Rightarrow \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) \div \sin\theta$$

$$= \frac{1-\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta}$$

$$= \frac{1-\cos\theta}{\sin^2\theta}$$

$$= \frac{1-\cos\theta}{1-\cos^2\theta}$$

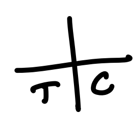
$$= \frac{\cancel{1-\cos\theta}}{\cancel{(1-\cos\theta)}(1+\cos\theta)}$$

$$= \frac{1}{1+\cos\theta} = R.H.S$$

(ii) Hence solve $\frac{cosec\,\theta - cot\,\theta}{sin\,\theta} = \frac{5}{2}$ for $180° < \theta < 360°$. [2]

$$\frac{1}{1+\cos\theta} = \frac{5}{2}$$

$$1+\cos\theta = 2/5$$



$$\cos\theta = -\frac{3}{5}$$

$$\theta = \cos^{-1}(3/5)$$

$$= 53.1$$

For negative,

$$\theta = 180° + 53.1°$$

$$= 233.1°$$

(b) Solve $tan\,(3\phi - 4) = -\frac{1}{2}$ for $0 \le \phi \le \frac{\pi}{2}$ $radians.$

[3]

$0 \le 3\phi \le \frac{3\pi}{2}$

$-4 \le 3\phi - 4 \le \frac{3\pi}{2} - 4$

(0.712)

$3\phi - 4 = \tan^{-1}(\frac{1}{2})$

$= 0.464$

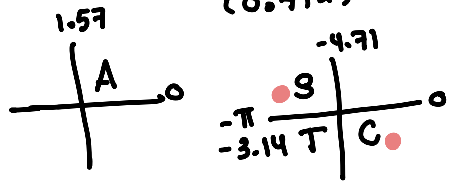
For negative,

$3\phi - 4 = -0.464, -\pi - 0.464$

$= -0.464, -3.606$

$3\phi = 3.536, 0.394$

$\phi = 1.18, 0.131$



4. (a) Solve $6sin^2x - 13\,cosx = 1$ for $0° \le x \le 360°$. [4]

$6(1-\cos^2 x) - 13\cos x = 1$

$6 - 6\cos^2 x - 13\cos x - 1 = 0$

$6\cos^2 x + 13\cos x - 5 = 0$

$(3\cos x - 1)(2\cos x + 5) = 0$

$\cos x = \frac{1}{3}$ or $\cos x = -\frac{5}{2}$ (reject)

$x = \cos^{-1}(1/3)$

$= 70.5°, 360° - 70.5$

$= 70.5°, 289.5°$

(b) (i) Show that, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\frac{4\tan y}{\sqrt{1+\tan^2 y}}$ can be written in the form $a\sin y$, where *a* is an integer.

[3]

$$\frac{4\tan y}{\sqrt{\sec^2 y}} = 4\,\frac{\tan y}{\sec y}$$

$$= 4\,\frac{\sin y}{\cos y} \times \cos y$$

$$= 4\sin y$$

$4\sin y$

(ii) Hence, solve $\frac{4\tan y}{\sqrt{1+\tan^2 y}} + 3 = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2}$ *radians*.

[1]

$$4\sin y + 3 = 0$$

$$4\sin y = -3$$

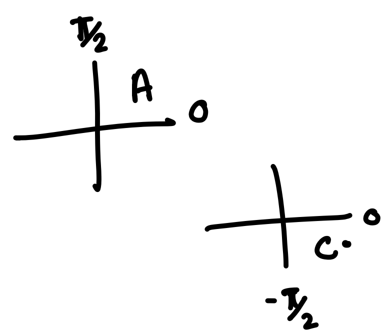
$$\sin y = -\frac{3}{4}$$

$$y = \sin^{-1}\left(\tfrac{3}{4}\right)$$

$$= 0.848$$

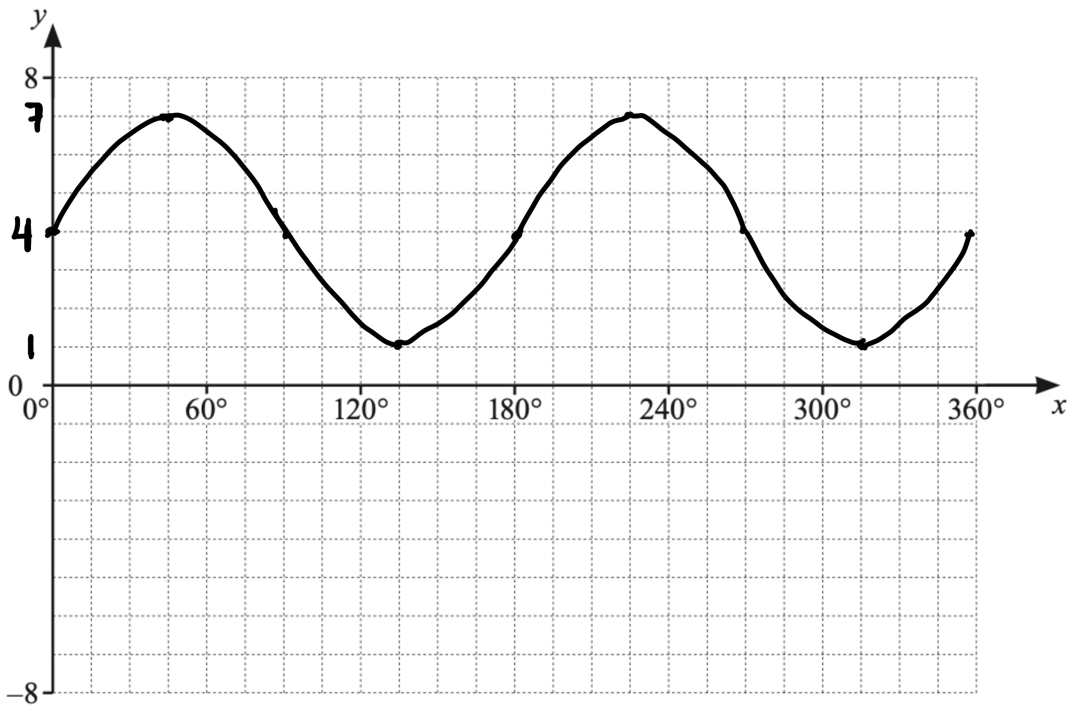
For negative,

$$y = -0.848$$



5. The function f is defined, for $0° \leq x \leq 360°$,by $f(x) = 4 + 3sin\,2x$.

(i) Sketch the graph of y = f(x) on the axes below.



[3]

(ii) State the period of f.

180°

[1]

(iii) State the amplitude of f.

3

[1]

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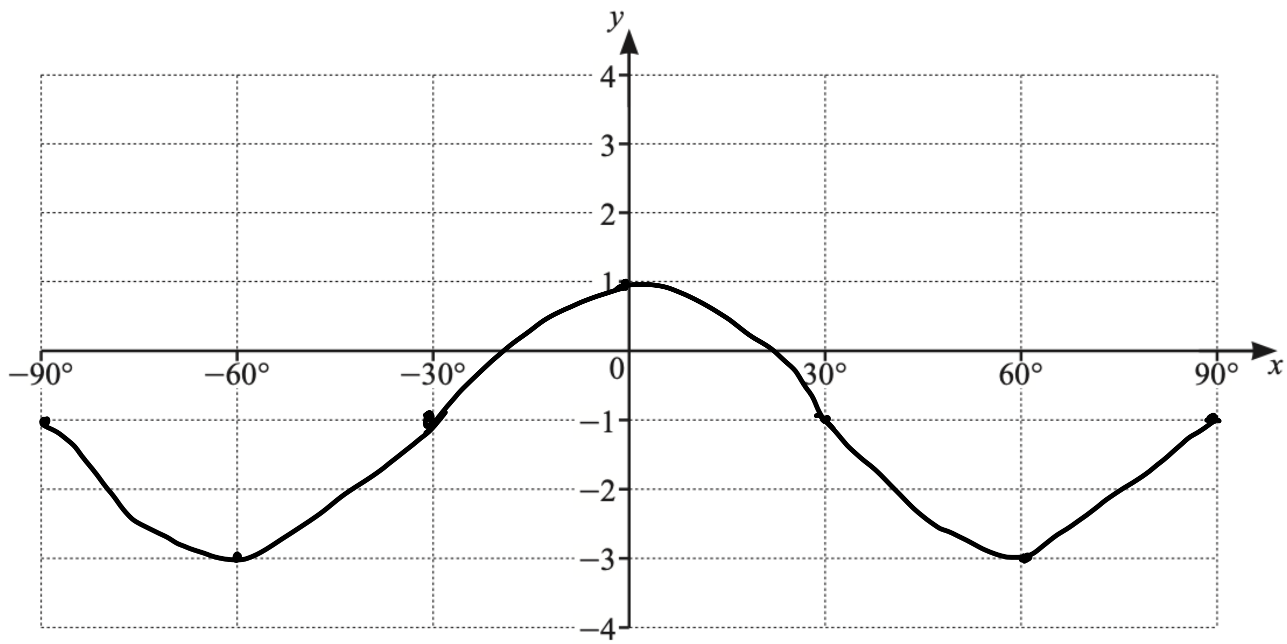
6. (i) On the axes below, sketch the graph of $y = 2cos\,3x - 1$ for $-90° \leq x \leq 90°$.



[3]

(ii) Write down the amplitude of $2cos\,3x - 1$.

[1]

2

(iii) Write down the period of $2cos\,3x - 1$.

120°

[1]

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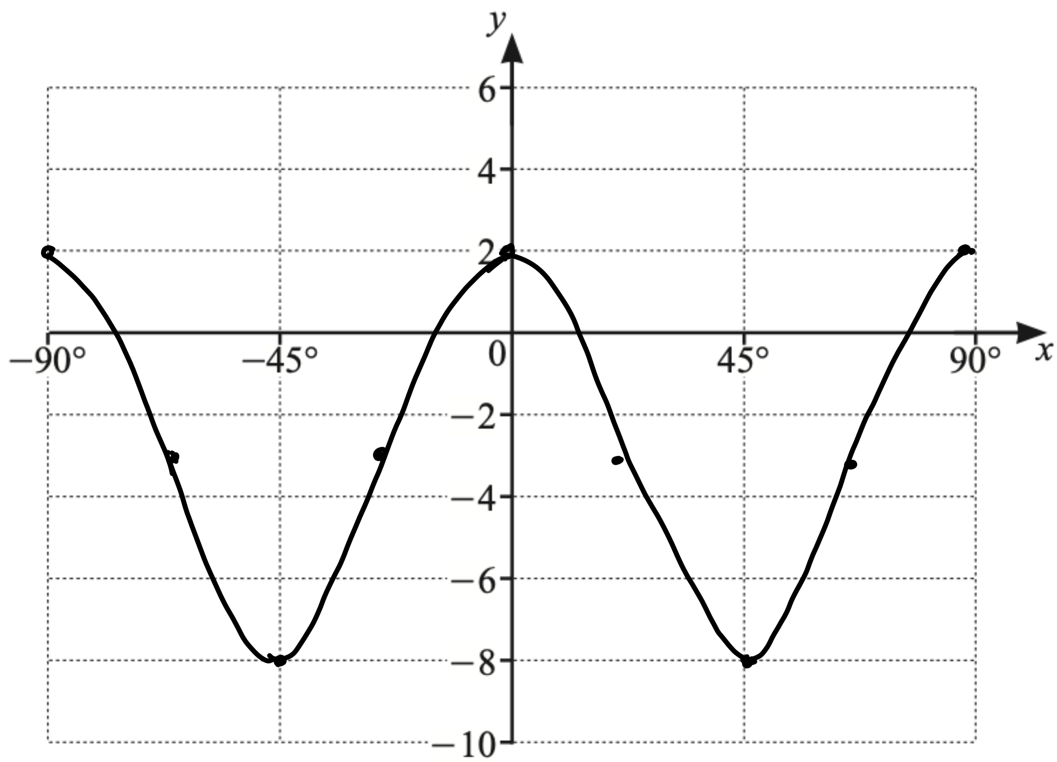
7. (i) On the axes below, sketch the graph of $y = 5cos\,4x\ - 3$ for $-\ 90° \le x \le 90°$.



[4]

(ii) Write down the amplitude of *y*.

5

[1]

(iii) Write down the period of *y*.

90°

[1]

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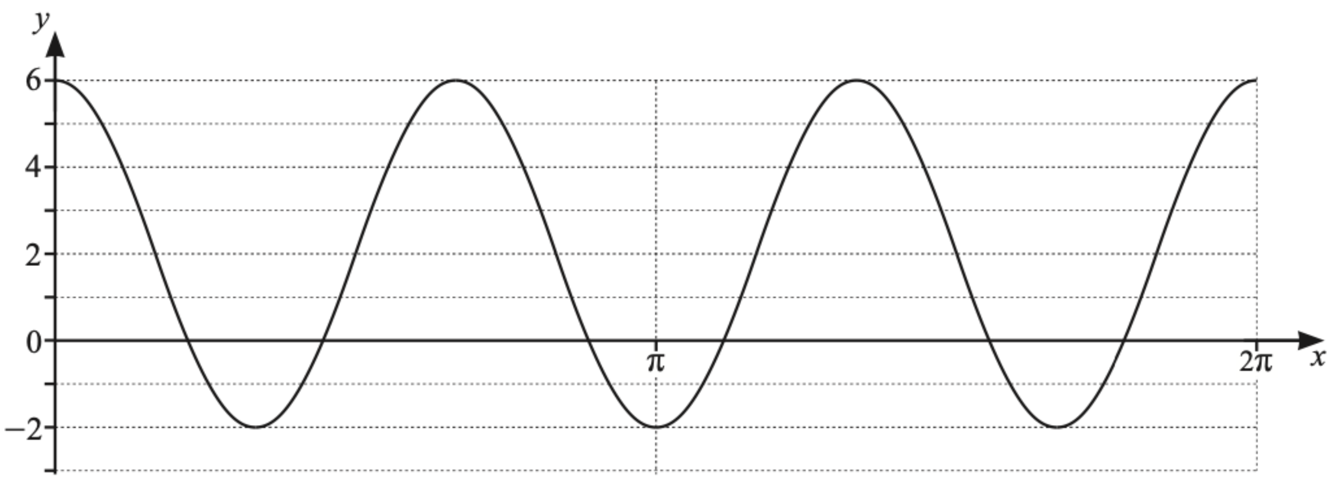
8.



The figure shows part of the graph of $y = p + q\cos rx$. Find the value of each of the integers p, q and r.

$p = 2$ $\quad\quad$ $q = 4$ $\quad\quad$ $r = 3$

[3]

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9. (i) Show that $\underset{\times \tan}{\dfrac{\tan x}{1+\sec x}} + \underset{\times 1+\sec x}{\dfrac{1+\sec x}{\tan x}} = \dfrac{2}{\sin x}$.

[5]

$$\text{L.H.S} = \frac{\tan^2 x + (1+\sec x)^2}{\tan x\,(1+\sec x)}$$

$1+\tan^2 x = \sec^2 x$

$$= \frac{\tan^2 x + 1 + 2\sec x + \sec^2 x}{\tan x\,(1+\sec x)}$$

$$= \frac{\sec^2 x + 2\sec x + \sec^2 x}{\tan x\,(1+\sec x)}$$

$$= \frac{2\sec^2 x + 2\sec x}{\tan x\,(1+\sec x)} = \frac{2\sec x\,\cancel{(\sec x+1)}}{\tan x\,\cancel{(\sec x+1)}}$$

$$= 2\sec x \times \frac{\cos x}{\sin x}$$

$$= 2\,\frac{1}{\cancel{\cos x}} \times \frac{\cancel{\cos x}}{\sin x}$$

$$= \frac{2}{\sin x} \quad \text{(R.H.S)}$$

(ii) Hence solve the equation $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 1 + 3sinx$ for $0° \leq x \leq 180°$.

[4]

$$\frac{2}{\sin x} = 1 + 3\sin x$$

$$\frac{2}{\sin x} - 3\sin x - 1 = 0$$

$$2 - 3\sin^2 x - \sin x = 0$$

$$3\sin^2 x + \sin x - 2 = 0$$

$$(3\sin x - 2)(\sin x + 1) = 0$$

$\sin x = \frac{2}{3}$ or $\sin x = -1$ (reject)

$$x = \sin^{-1}(2/3)$$

$$= 41.8, 180° - 41.8$$

$$= 41.8°, 138.2°$$

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10. (i) Show that $\frac{cosec\,x - cot\,x}{1 - cos\,x} = cosec\,x$. [3]

$\frac{1}{\sin x} - \frac{\cos x}{\sin x} \times \frac{1}{1-\cos x}$

$= \frac{1-\cancel{\cos x}}{\sin x} \times \frac{1}{1-\cancel{\cos x}}$

$= \frac{1}{\sin x} = \operatorname{cosec} x$ (R.H.S)

(ii) Hence solve $\frac{cosec\,x - cot\,x}{1 - cos\,x} = 2$ for $0° < x < 180°$. [2]

$\operatorname{cosec} x = 2$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}(1/2)$

$= 30°, 180°-30$

$= 30°, 150°$